

IMPACT OF ENGINEERING MODELING ASSUMPTIONS ON ASSESSING SEISMIC RESISTANCE OF MONTGOMERY BLOCK BUILDING

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ABSTRACT: The Montgomery Block building is used as a case study to assess the effectiveness of conventional analysis methods commonly used by practicing engineers throughout North America when conducting seismic evaluation work to predict the seismic performance of unreinforced masonry (URM) buildings. This building predominantly constructed of URM pier-and-spandrel facades and URM shear walls was built in 1853 and survived the 1906 San Francisco Earthquake despite being roughly 15 km from the San Andreas Fault that ruptured in 1906. It is found that the impact of engineering modeling assumptions can be of nearly an order of magnitude for buildings of that type, depending on the structural model selected by the engineer. Of all the conventional modeling procedures considered in this paper, it is found that only the special procedure of the *Uniform Code for Building Conservation* (UCBC) comes close to predicting the observed seismic response. However, some extensions of the more conventional modeling procedures are useful in demonstrating the potentially beneficial period elongation that accompanies structural strength degradation as damage progresses.

INTRODUCTION

Recent North American earthquakes have generated a sudden interest in the seismic evaluation and rehabilitation of existing unreinforced masonry (URM) buildings in eastern North America. These buildings, constructed in the absence of mandatory earthquake design requirements, can be vulnerable to earthquakes. However, unlike California, most of eastern North America is without the benefit of an "umbrella" seismic retrofit ordinance clearly mandating how and on which basis existing URM buildings must be evaluated, leaving this responsibility to individual owners and their engineers. Consequently, there currently exists a striking variability in the analytical techniques adopted by structural engineers when dealing with the seismic issues of URM buildings east of the Rocky Mountains. The eastern engineering community is split between those who rely on conservative models and analytical techniques, advocates of the new special codified procedure for the seismic evaluation of URM buildings, and, to a lesser extent, proponents of the finite element analysis procedure. Finally, there are those structural engineers who outright refuse to work with existing URM buildings, convinced that their seismic resistance is essentially null and that reliable retrofit methods are inconceivable. Not surprisingly, some owners have expressed concerns that engineers using modern structural analysis models could actually be incapable of making accurate assessments of the true seismic resistance of such existing buildings.

Clearly, a better understanding of the seismic behavior of URM buildings can undoubtedly translate into savings when seismic retrofit is considered or even demonstrate in some cases that a building is already capable of sustaining a broad range of earthquake ground motions. This is particularly important for vintage and historical structures for which it is always desirable to minimize the extent of structural alterations. Because many eastern American practicing engineers

have so far favored conservatism when assessing the seismic resistance of URM buildings, it is worthwhile to examine whether various common modeling procedures can provide reliable engineering assessment of seismic survivability and whether the resistances calculated using these models differ sufficiently to be of practical significance.

For that purpose, a URM building known to have survived a very large earthquake was chosen to investigate which conventional analytical methods could best "predict" this seismic survival and simultaneously gauge the impact of common engineering modeling assumptions on this assessment of seismic performance. In other words, this is equivalent to looking at how a structural engineer could have calculated the potential seismic resistance of this building before the 1906 earthquake if he/she had enjoyed the benefit of today's practice and tools and then reviewing how this assessment compares with the actual observed seismic performance.

Because the San Francisco Earthquake of 1906 of Richter magnitude 8.3 is perceived by most eastern North American engineers to be the most severe ever to hit the continental United States (although the 1811–1812 New Madrid earthquakes may actually have been larger, the 1906 earthquake is certainly not a trivial earthquake), a URM building survivor of this earthquake, the Montgomery Block building, built in 1853, was selected for this study. This office building survived the 1906 tremor without damage despite being roughly 15 km from the ruptured San Andreas Fault (Freeman 1932).

In this paper, following a description of the Montgomery Block building and presentation of material properties used, the maximum seismic capacity of this building is calculated using a number of different modeling procedures commonly used by practicing engineers in seismic evaluation work (note that while numerous advanced analytical modeling techniques for URM buildings exist and have been proposed in the existing literature, only those currently used on a regular basis by engineers throughout North America are considered in this study). Results are expressed in terms of base shears under statically applied lateral loads. Then, these calculated base-shear capacities are compared against estimated linear-elastic response spectra of the demand (i.e., strength requirement) likely produced by the 1906 San Francisco earthquake that hit this building.

The professional, legal, and other nontechnical impacts ensuing from the selection of particular analytical models, although equally important, are not addressed here, but the findings of this paper would hopefully provide useful material for these other debates.

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DESCRIPTION OF MONTGOMERY BLOCK BUILDING

Many URM buildings in San Francisco are known to have survived the 1906 earthquake without damage. This has been well documented in various documents, including Freeman's (1932) book written for a target audience of insurance executives, property owners, and structural engineers. However, when the first writer tried to obtain drawings for many of these buildings, he found that owners of these still-existing survivor buildings are reluctant to release any engineering information to be used in seismic-resistance assessments (for reasons that can be guessed, given the foregoing introduction), even when solicited through their usual structural consultant. The Montgomery Block building was therefore selected for the following reasons: (1) It has survived the 1906 San Francisco earthquake, as reported by Freeman (1932); (2) good quality archive drawings were available (Cardwell et al. 1958); and (3) this structure did not have a "protective" owner because it was demolished approximately 25 years ago to be replaced by the "Transamerica Pyramid" high-rise (thus, no risk of litigation). The fairly regular structural layout of the building also proved to be an asset for this case study.

The four-story Montgomery Block building was approximately 37×42 m in floor plan. Its first story was relatively more rigid than the other three with identical floor plans because more URM shear walls were present in both perpendicular directions at that story. The thickness of all walls and piers of this structure was 0.45 m. The heights of the first, second, third, and fourth stories were, respectively, equal to 3.8, 3.5, 3.2, and 2.9 m. All the floors and the low attic ceiling of the

last floor, which had a height of 0.75 m, were made of wood. This building had an interior court providing natural light, hence surrounded by walls with numerous window openings. Three of the exterior facades were equally perforated walls; the north, south, and west facades were situated on Washington, Merchant, and Montgomery streets, respectively. All perforated facades had 1.2-m-deep URM spandrel beams on the first three stories and 1.7-m-deep beams on the fourth story. All piers were continuous from base to roof. The east wall was nearly solid with only a few small openings (such as for fire exits and other access to a back alley). Examples of the reconstructed architectural drawings taken from microfilm are presented in Figs. 1 and 2. Although some piers were shown discontinuous at the first story on those 1958 drawings, an earlier photograph of that building (Kirker 1973) reveals that all piers on the facade were originally continuous from roof to foundation with regular-size 0.9-m-wide openings throughout. This original facade layout was adopted for this study.

For URM buildings of that era, it is reasonable to assume a flexible wood diaphragm; this implies that the lateral-load-resisting structural elements must resist the seismically induced horizontal forces attributable to their tributary area. In this case, the heavily perforated wall facades are the weakest structural elements. Further, in a survey of all tributary areas and corresponding lateral-load-resistant structural elements present in the Montgomery Block building, the weakest wall of this building was identified to be (seismically speaking) the perforated URM west facade wall. Hence, this paper focuses on the seismic resistance of that facade wall.

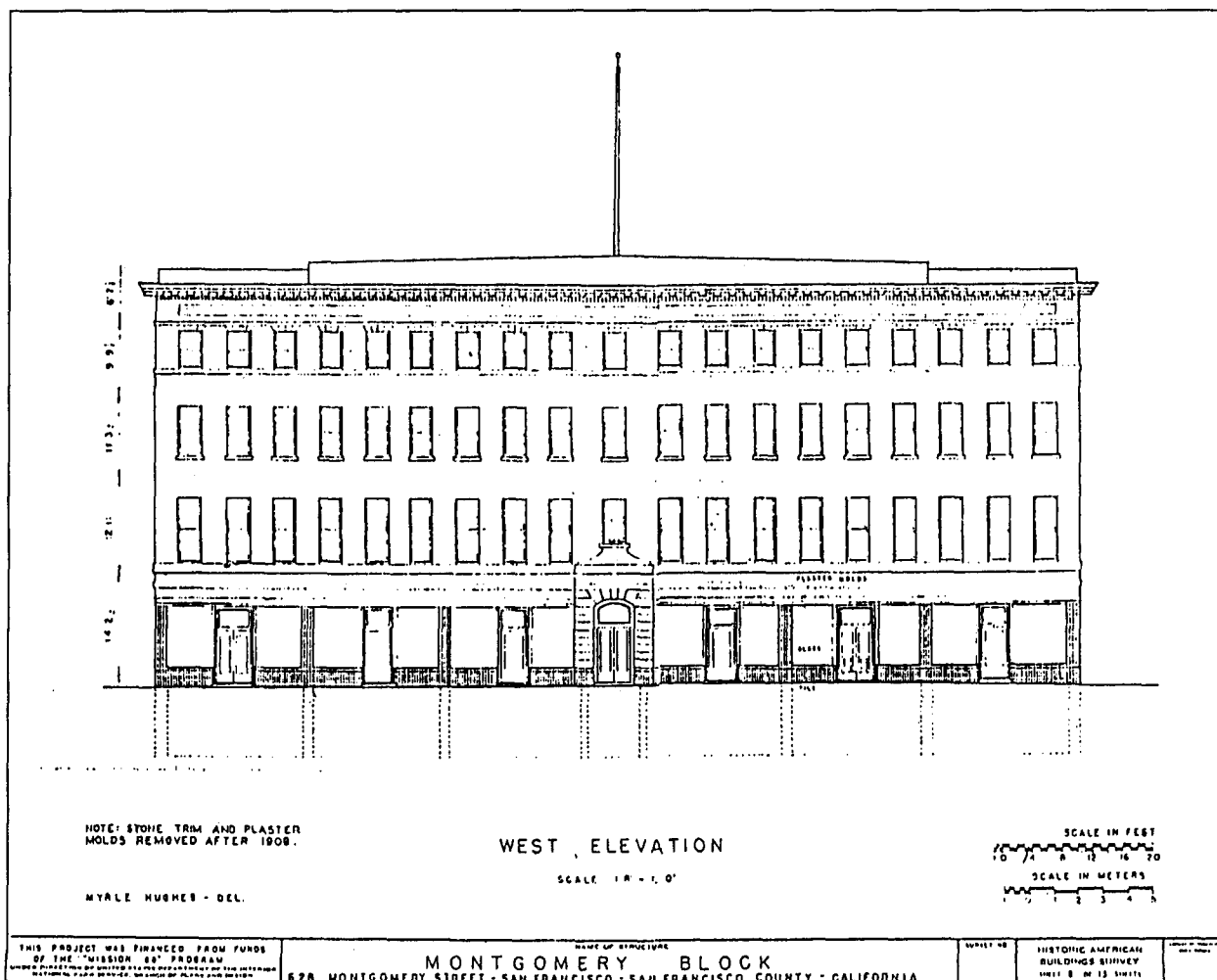


FIG. 1. West Facade Elevation of Montgomery Block Building (Cardwell et al. 1958)

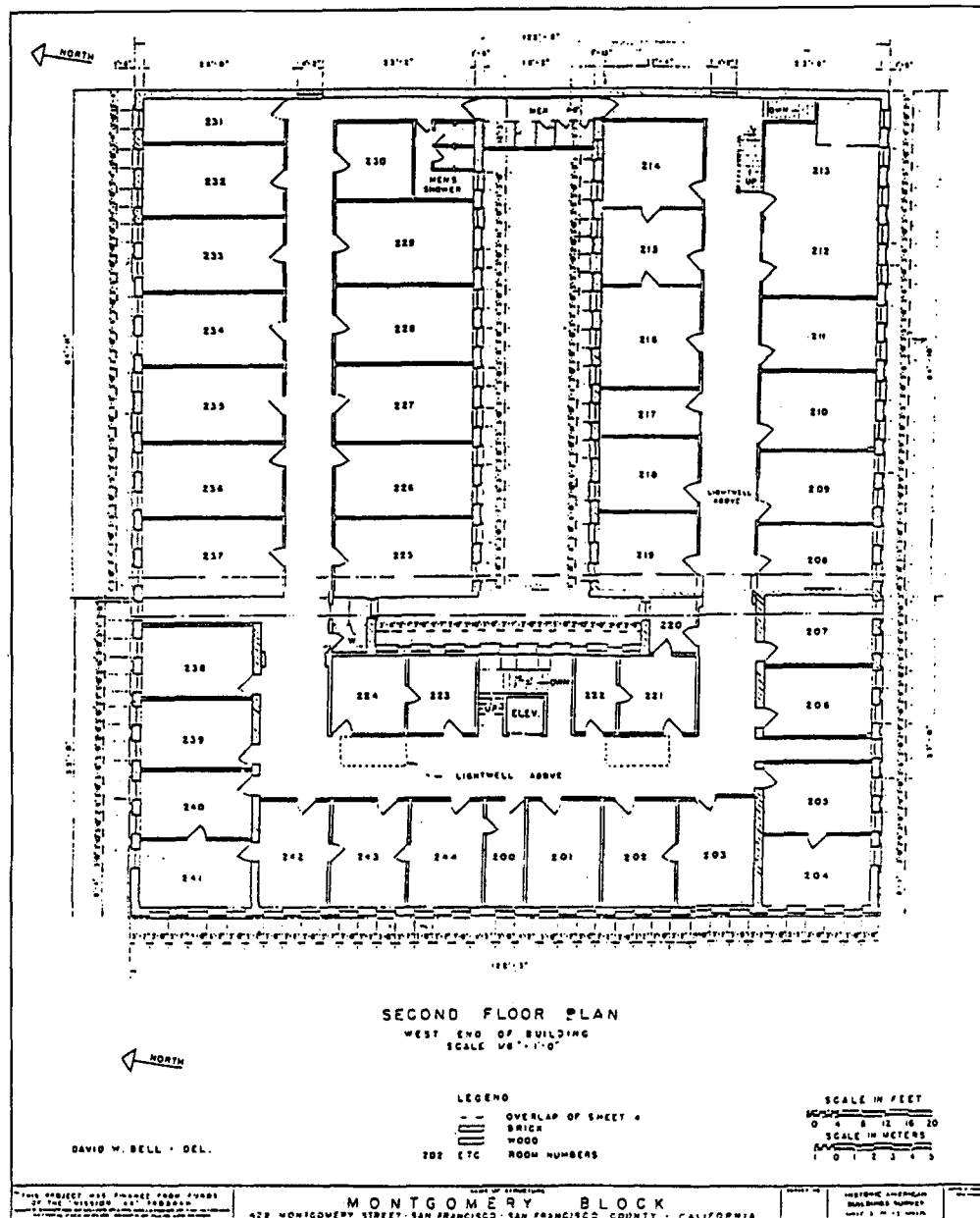


FIG. 2. Second-Floor Plan Montgomery Block Building (Adapted, Cardwell et al. 1958)

MATERIAL CHARACTERISTICS OF MONTGOMERY BLOCK BUILDING

Obviously, because the Montgomery Block building does not exist anymore, on-site measurements of material properties were not possible. Therefore, the properties needed for this study were chosen as representative of those often reported in the existing literature for old Californian URM buildings (ABK 1984; Epperson and Abrams 1990; Priestley 1985), and equal to

- Young modulus: $E_m = 1,000$ MPa
- Allowable flexural tensile stress: $f_t = 0.19$ MPa
- Allowable flexural compressive stress: $f_c = 2.6$ MPa
- Allowable shear stress: $f_s = 0.24$ MPa

These are compatible with the types O, K, or N mortars, which were most commonly used along with brick units having typical strength of 20–40 MPa before 1940. For simplicity, no distinction is made between tensile capacity for tension normal to the bed joints (as in piers) and normal to the head joints

(as in spandrels), although some differences in the tensile failure behavior of these structural elements exist as reported by Abu-El-Magd and MacLeod (1980). The effect of axial compression in enhancing the shear capacity of masonry has been considered as indicated in a following section. Also, the use of equivalent linear relationships for many properties known to be nonlinear near the ultimate was deemed to be of minor significance in this study. This implies that none of the procedures considered here account for possible ductility or energy dissipation of masonry structures at the material level; this is consistent with what is typically done by practicing engineers at this time.

As will become evident later, only the flexural tensile value in the foregoing list of properties has an impact on the findings in this study. It has been taken as liberally high as realistically possible to ensure that predicted seismic resistances reported here are optimistic.

Allowable values have been used here to calculate and compare the seismic capacities obtained using various procedures in compliance or inspired from code requirements. However, although this closely followed a “practicing engineer” ap-

proach, it created a slight dilemma because the true ultimate seismic capacity and behavior of the buildings were equally of interest here. Hence, to resolve this matter, although member-strength calculations used allowable stress values, the resulting seismic resistance of the entire building (i.e., maximum base-shear capacity) is scaled upward by an appropriate factor, in a later section, when the results are interpreted against the observed seismic performance. Therefore, for the purpose of member-strength calculations, when an element was found to be overstressed using allowable stress calculations, it was treated as "cracked" and "failed." This is important because in some of the models used in this study, as seen in subsequent sections, the physical properties of cracked elements are modified to account for this effect prior to reanalysis of the structure. Therefore, the words "crack," "cracking," "failure," and "rupture," for elements, are substituted liberally to the corresponding "overstress," (and synonyms) which would normally be used in working stress analyses, with the understanding that the structural capacities of the entire building thus obtained are corrected appropriately in a later section. Although this approach can be challenged in some ways, the reader will eventually realize that it does not have a sizeable impact on the conclusions reached in this study.

MODELING PROCEDURES CONSIDERED

Three modeling approaches are considered in this study and are described. However, it must be understood that the first two procedures adopted are actually based on conventional design practice. The obvious difference between design and evaluation methods is recognized by the writers: design methods appropriately rely on conservative assumptions and adopt simple models to be able to design safe structures efficiently and economically, whereas in principle, evaluation methods should be more rigorous and precise because it is based on the as-built conditions and actual material properties versus nominal. However, although it may be easy to argue that design methods are inappropriate for evaluation, the fact remains that many structural engineers throughout eastern North America still use those design methods when evaluating URM buildings. This philosophy of "code-compliance" is usually adopted for the legal protection it apparently provides. However, in this study, these simple modeling procedures are extended as far as the authors believed a liberal but code-compliant practicing engineer would go using these design methods in an evaluation perspective, to provide an appreciation of postcracking structural behavior.

It is noteworthy that, for this study, equivalent earthquake static loads were applied along the height of the buildings in compliance with the distribution prescribed by most building codes [e.g., National Research Council of Canada (1990) and *Uniform Building Code* (UBC) (1994a)], except for the *Uniform Code for Building Conservation* (UCBC) special procedure described in a later section for which a special vertical load distribution is mandated [see Bruneau (1994b) and UCBC (1994)]. Also, for all analyses except the UCBC special pro-

cedure, the facade has been modeled by using the structural analysis program SAP90 (Wilson and Habibullah 1990). The spandrels' axial deformations were assumed negligible; all piers were constrained to have the same lateral floor displacements. Tributary area concepts were used to calculate and distribute the floor gravity loads on the west facade, and the calculated self weights of masonry piers and spandrels were applied on the respective individual piers at each level. Also, in view of the short distance between the lower and upper parts of the low-ceiling attic of this building, the entire roof mass was lumped at an equivalent height. The resulting calculated lumped masses for the entire building are presented in Table 1. It is noteworthy that the contribution of wood to the total structural floor mass varies from 34 to 23% from the upper to the lower floor. The larger number of masonry walls and piers on the first floor and the low-ceiling attic on the last floor explain this variation.

Piers-Only Model

A conservative model sometimes adopted by practicing engineers assumes that the spandrel beams will crack under a very low lateral load, leaving the piers alone to resist the lateral loads. This approach is not unlike that recommended by some researchers for the analysis of reinforced masonry walls having numerous opening, where the masonry above and below the openings is neglected (Englekirk and Hart 1984). Although this models the structure at its ultimate state if the spandrels are shallow or not well connected to the piers, it immediately assumes a structure in its degraded condition, neglecting the potentially larger capacity of the structure before cracking.

There are 20 piers in the facade of interest here: two are 1.3 m wide, 17 are 1.0 m wide, and one is 0.9 m wide. Element failure by flexural cracking, shear cracking (diagonal tension), or compression crushing has been considered. A number of models of this behavior are suggested in the literature [Mayes and Clough (1975); Canadian Standards Association (CSA) (1984); ASCE ("Building" 1988); Bruneau (1994a); UBC (1994a)]. In this study, failure of a pier or spandrel is defined by a first-cracking criteria, i.e., deemed to occur at the attainment of allowable tensile or shear stresses in this element. Also, two shear resistances of the masonry have been considered: in the first case, the effect of axial compression in enhancing this shear resistance has been neglected (pure shear case), and in the second case, it has been considered (precompressed shear case). The elementary equations of mechanics of materials typically integrated in most design codes and standards are used for that purpose.

Typically, practicing engineers who use the piers-only model have also adopted the conservative assumption that piers failed in tension may not be relied upon anymore to resist compression or shear. In that perspective, analysis must stop at the discovery of first-pier cracking. However, numerous experiments have demonstrated that URM piers can rock and safely resist gravity loads in spite of the presence of through-

TABLE 1. Lumped Masses: Contribution from Various Components for Entire Montgomery Block Building

Story No. (1)	LUMPED MASSES				Total (kg) (6)
	Wood (kg)		Masonry (kg)		
	Partition walls (2)	Floor (3)	Spandrels (4)	Piers and walls (5)	
4	42,856 (8%)	132,282 (26%)	111,381 (22%)	220,000 (44%)	506,519 (100%)
3	90,145 (12%)	148,549 (19%)	81,005 (10%)	463,009 (59%)	782,708 (100%)
2	99,011 (12%)	155,371 (18%)	81,005 (10%)	508,624 (60%)	844,011 (100%)
1	70,933 (7%)	162,495 (16%)	108,005 (10%)	703,348 (67%)	1,044,781 (100%)

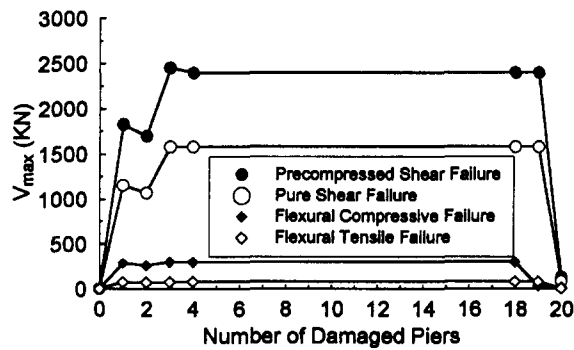


FIG. 3. West Facade's Lateral-Load Capacity (2D Analysis, Piers-Only Model)

cracks [e.g., Epperson and Abrams (1990); Prawel and Lee (1990); and Costley and Abrams (1995)]. In fact, this safe rocking mechanism recognized by the special procedure of the UCBC, and many engineers are now familiar with this reality. Therefore, to provide some insight into structural behavior beyond first cracking and investigate where conventional models would lead if used by practicing engineers who do not consider first cracking of URM to be the structure's failure criterion, a multistep analysis was conducted by eliminating from the structural model the lateral-load resistance of piers whose capacity has been reached (while preserving their gravity carrying capacity) and reanalyzing the remaining structure. Computer modeling procedures make it possible to numerically express the foregoing phenomenon simply by assigning low numerical values to the physical properties affecting lateral-load resistance of piers that have cracked, without affecting axial properties.

Although there is no evidence to support that the use of such a multistep analysis is justified when shear and/or compression failures occur instead of tension failure, for completeness (and academic interest), this procedure was nonetheless followed for the four possible failure modes mentioned in the foregoing section and considered individually. Results are shown in Fig. 3 in terms of base-shear capacity V_{max} as a function of number of failed piers. The horizontal axis is expressed as a continuum, with the capacity varying linearly between points at which another pier fails. Clearly, the governing failure mode is flexural tension, as may be expected here because the piers are modeled as tall cantilevers. However, it is observed that the maximum base-shear capacity is reached beyond first cracking. In fact, two piers stiffer than all the others attracted more force and eventually cracked first, but the 18 remaining nearly identical piers had sufficient reserve capacity to absorb this loss and even carry a higher applied load thereafter. A maximum base shear of 77 kN was attained at the third analysis step, prior to cracking of the 17 identical piers.

For the nongoverning failure modes, the calculated ultimate base-shear capacities were 295 kN for flexural compression and 1,579 and 2,453 kN for pure shear and precompressed shear, respectively, i.e., considerably more than obtained by flexural tension.

Frame Models

If the spandrels are deep and/or of short span, both the piers and the spandrels may fail, in any sequence. Thus, to address this possibility, the facades are then modeled as rigid frames. Here, again, most engineers who would use this model would not rely on strength estimates beyond that obtained when first cracking is reached anywhere in the structure, because of concerns regarding the shear and compressive resistance of cracked piers as described in the previous section. However, for the same reasons noted earlier, multistep analysis was con-

ducted considering the same four possible failure modes, but here only the case giving least capacity was retained for the following presentation. When the capacity of an element was reached, its initial flexural inertia and shear area were replaced by arbitrarily small values in the computer model such that its lateral-load resistance was eliminated effectively while retaining its axial-load-carrying capacity, all without making the structural computer model unstable locally.

Basic Frame Model

As in the previous model, for all analyses, flexural tensile failure consistently governed. A maximum base-shear capacity V_{max} of 171.5 kN was attained at first cracking but a collapse failure mechanism (general failure) only occurred when all piers of the last floor cracked, after 80 analysis steps. Fig. 4 illustrates the evolution of V_{max} with the number of damaged elements (refer to the curve labeled "without rigid offsets" for reasons described in the next section). The dashed line is obtained by calculating the new capacity of the remaining structure immediately following elimination of the lateral-load resistance of a cracked element. By connecting all peak values with a curve of constantly decreasing strength, an assessment of the resulting progressive strength degradation is possible, as illustrated by the dotted line. Nonetheless, it is clearly seen in Fig. 4 that the calculated ultimate strength for the frame model is equal to the strength at first cracking.

The corresponding evolution of the facade's fundamental period of vibration, as a function of the number of damaged elements, also was obtained and is presented in Fig. 5; it is noteworthy that the west facade's initial fundamental period was 0.5 s, and that it increased throughout the multistep analysis process.

Frame Model Considering Rigid Offsets at Joints

A second frame model of the west facade was constructed, this time considering the finite dimensions of the very rigid

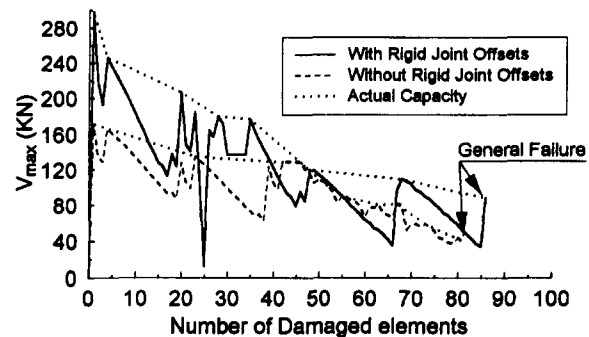


FIG. 4. West Facade's Lateral-Load Capacity (2D Analysis, Frame Models)

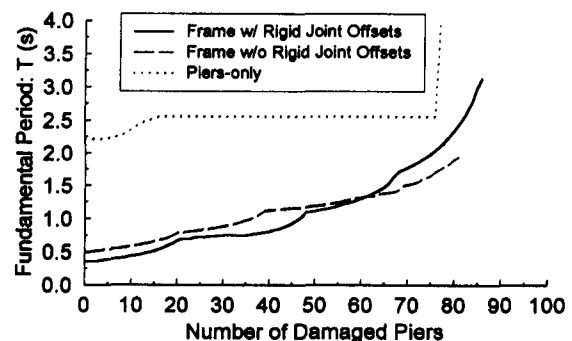


FIG. 5. West Facade's Fundamental Period (2D Analysis, All Models)

spandrel-pier joints. Structural analysis computer programs usually model this condition using rigid offsets at the ends of structural members (Wilson and Habibullah 1990). These rigid offsets are the distances from the intersection points of centerlines to the faces of the piers and spandrels. Their introduction in the model results in shorter flexible length of the members because the actual elements' lengths are reduced by a quantity equal to the sum of their rigid-ends lengths. Moreover, all the member forces are calculated and checked against resistance at the outer ends of the rigid offsets (faces of supports), as logically expected.

The same multistep analysis described for the foregoing model was conducted. The maximum base-shear response also was attained here at first cracking, with a V_{max} of 297.8 kN. A failure mechanism was attained, at the 84th analysis step, when all the piers of the first floor failed. The resulting capacities and periods as a function of the corresponding number of damaged elements are presented in Figs. 4 and 5. In this case, the initial period was 0.35 s. At the last analysis step, this value became 3.12 s, exceeding the maximum of 1.94 s reached when rigid offsets were neglected.

An investigation of the lateral floor displacement Δ was also made for each floor of the facade. When V_{max} was 297.8 kN, the floor displacements were 1.64, 2.53, 3.04, and 3.24 mm from the first to the fourth floor, respectively. Just prior to development of the failure mechanism, these lateral floor displacements reached 23.76, 61.40, 79.68, and 80.02 mm. These displacements were observed to increase faster as the number of damaged piers grew.

By combining results, a shear-displacement curve of the west facade was constructed, as shown in Fig. 6, for the frame with rigid offsets model. This reveals a strength and stiffness degrading behavior of the facade. The resulting area under the

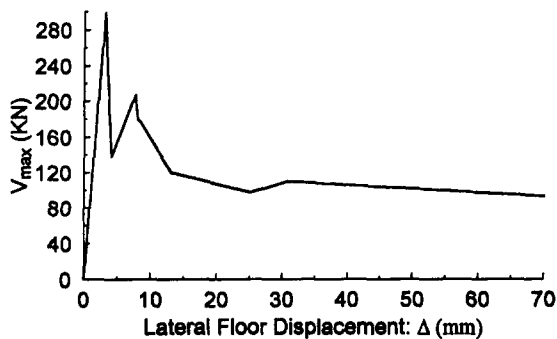


FIG. 6. Base Shear versus Fourth-Floor Lateral Displacement Behavior (2D Analysis, Frame Model with Rigid Offsets)

strength-deformation curve of Fig. 6 could be interpreted as a dissipated energy, and this URM building could be said to have some structural ductility under monotonically increasing static loading. However, the area under the shear-displacement curve obtained from a push-over analysis is known to be uncorrelated to dynamic hysteretic energy dissipation (Lawson et al. 1994), and the reader should not hastily extrapolate from the above. Also, the energy released by the cracking phenomena itself is neglected here.

Comparison of Frame Models Results

The following number of worthwhile observations can be made when comparing the results obtained for the west facade of the Montgomery Block building using the piers-only and frames models:

- Greater lateral-load resistance is obtained when progressively more complex models are adopted. The base-shear capacity calculated by the frame with offsets model is nearly two and three times greater than those obtained by the frame without rigid offsets and piers-only models, respectively.
- In terms of base shears normalized by reactive weight (i.e., V_{max}/W ratios), the calculated base-shear resistances are relatively low, increasing from 2.1% for the piers-only model to 4.7% for the frame without rigid offsets model and 8.1% for the frame with rigid offsets model.
- For all models, the facade's fundamental period increases as damage progresses. This translates into a progressive lateral softening of the building. However, the rate of this softening varies for the various models. For the frame models, this period elongation is considerable, reaching more than three to eight times the initial value prior to the development of a collapse failure mechanism.
- For the frame models, the lateral floor displacements increase nearly exponentially with the number of damaged piers (Boussabah 1993). By contrast, in the piers-only model, the roof displacement is 75 mm at first cracking and 77 mm at maximum capacity, i.e., actual softening of lateral stiffness is minimal.
- Adopting different modeling assumptions yields different final damage patterns and collapse mechanisms. Fig. 7 illustrates the evolution of damage in both frame models investigated. Indeed, a collapse mechanism occurred in different stories in the two frame models. Moreover, many spandrels were undamaged when the collapse mechanism occurred. Therefore, to assume that all span-

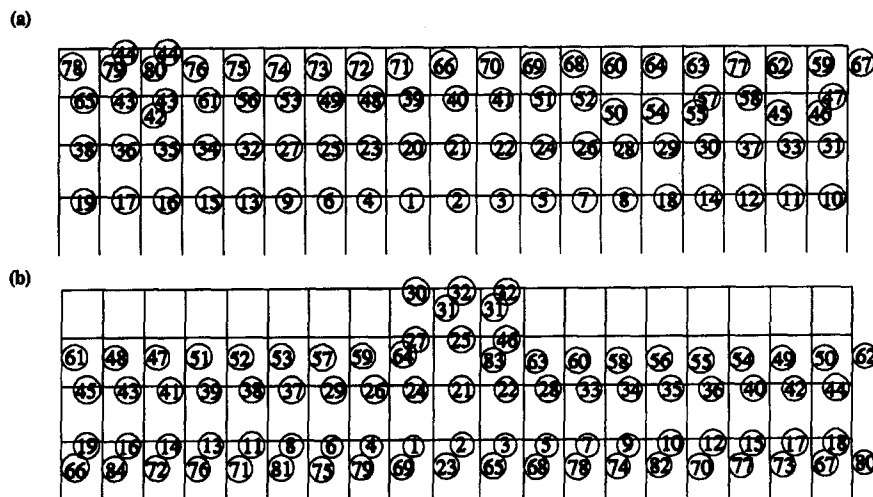


FIG. 7. Damage Evolution In: (a) Frame without Rigid Offsets; (b) Frame with Rigid Offset

drels will crack first, as done in the piers-only model, appears to be a wrong assumption.

The allowable flexural tensile stress in URM elements would need to be unrealistically large to preclude tensile failure as a governing mode for this west facade. However, if the facade could be reinforced to eliminate the problem of tension failure, the next governing failure mode to occur would be pure shear, with maximum facade capacities of 1,012.4 and 826.0 kN for the frame models with and without rigid offsets, respectively, instead of the 297.8 and 171.5 kN obtained in the foregoing section. These capacities also would be obtained at first cracking.

UCBC Special Procedure

The 1994 edition of the UCBC (1994) is a code specifically developed to address the problems germane to existing buildings, which often cannot economically be upgraded to comply fully with modern codes. It includes provisions for the seismic evaluation and strengthening of URM-bearing wall buildings (Appendix Chapter 1), for which a special procedure based on empirical evidence and applicable to certain types of URM buildings is proposed, in California, particularly where endorsed by local ordinances mandating the mitigation of seismic hazards from URM buildings. These provisions have been used to investigate and retrofit, when necessary, thousands of buildings. Because of space constraints, a detailed review of all aspects and steps of the methodology is not possible here and the reader should refer to other sources [ABK 1984; Structural Engineers Association of Southern California (SEAOSC) 1981, 1983, 1986, 1991; Seismic Safety Commission 1990; Bruneau 1994b] if interested in the specifics of its application. It is noteworthy that the UCBC is written in a working stress

TABLE 2. Montgomery Block Building: Calculated Base-Shear Capacities of West Facade Wall as per UCBC's Special Procedure

Story No. (1)	West Facade (kN)			
	ΣF_{wx} (2)	ΣV_R (3)	ΣV_a (4)	α (5)
4	89	204	2,182	2.29
3	217	1,018	2,182	4.69
2	351	1,195	2,182	3.40
1	501	1,644	2,182	3.28

Note: Facade's capacity = 1,148 kN.

design format, but the special procedure was originally formulated (ABK 1984) and should be understood in terms of ultimate loads and behavior.

For the current study, it is only necessary to understand that the UCBC considers that URM piers can have two modes of in-plane behavior. A pier can either fail in shear, with typical "X" cracking, or rock back and forth as a solid block between rigid spandrels. The UCBC special procedure also assumes that ground motion is not amplified along the height of a URM building. Finally, the methodology relies on a minimum of structural integrity, and should anchors between URM walls and floor-roof diaphragms be missing or inadequate, the methodology automatically calls for a retrofit requiring at least the installation of a new wall anchorage system, including the bracing of parapets.

The UCBC procedure was used to determine both the in-plane ultimate lateral resistance of the west facade of the Montgomery Block building and the vertical distribution of the corresponding equivalent static seismic force. The west facade was modeled as a combination of piers spanning between thick rigid spandrels. At each level, the seismic shear forces (F_{wx}), the shear cracking strength (V_a), and the rocking restoring shear strength (V_R) were calculated in accordance with the UCBC; these are presented in Table 2. The adequacy of the structural system at each level was then checked according to the flowchart of Fig. 8 (SEAOSC 1991). In this case, the calculated rocking restoring shear strength (V_R) of every pier was found to be less than its corresponding cracking shear strengths (V_a) indicating a dominating rocking behavior. Moreover, at each floor, the UCBC-specified shear force demand ΣF_{wx} applied to the entire facade was observed to be less than the total restoring shear capacity (Table 2). By defining the ratio α of story capacity to total applied shear force and calculating this value at each story, it was possible to obtain the ultimate capacity of the entire facade by multiplying the total base-shear force by the smallest calculated α -value (Table 2). This minimum obtained α -ratio being 2.29 at the fourth floor, a corresponding calculated base-shear capacity of 1,148 kN was calculated.

It is noteworthy that the demand-capacity ratios of the roof and floor diaphragms, as well as slenderness ratios of all URM piers and walls, were also calculated and checked against the corresponding limits of the UCBC (1994). It was established that the diaphragm spans were acceptable without the presence of cross-walls and that out-of-plane dynamic stability existed for all walls and piers of this building [see ABK (1984); SEAOSC (1991); Bruneau (1994b); among many, for definition of dynamic stability], assuming the presence of proper

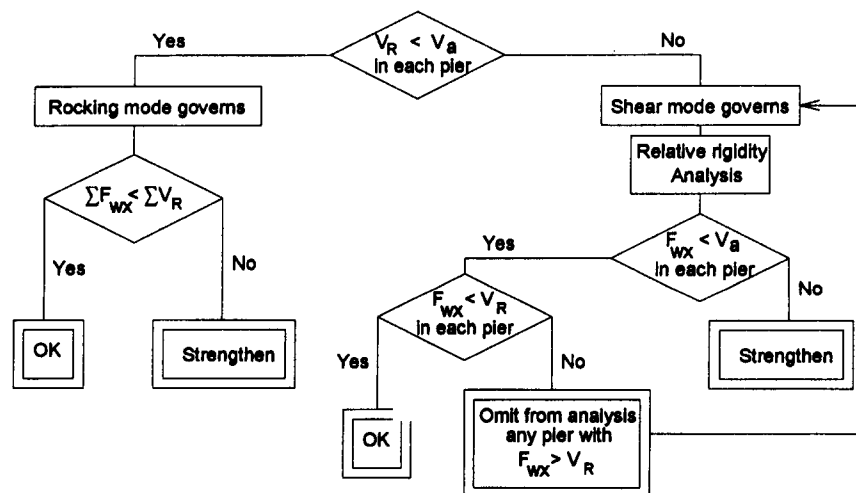


FIG. 8. Flowchart for Analysis of URM Walls for In-Plane Shear by UCBC Procedure

anchorage of walls to diaphragms. Unfortunately, the available drawings do not mention whether wall anchorage existed in this building.

Based on the UCBC evaluation, the west facade would not have required seismic rehabilitation beyond the provision of proper wall-to-diaphragm anchorage if found to be absent or deficient. However, this should not be construed as a prediction of no structural damage during a major earthquake. The UCBC special procedure is only intended as a risk-reduction measure (UCBC 1994).

INTERPRETATIONS OF RESULTS—SUPPLY VERSUS DEMAND

Ultimately, the adequacy of a modeling assumption is verified by its ability to explain successfully or demonstrate an observed behavior. Hence, it remains to be investigated as to which of the foregoing procedures could predict the Montgomery Block building's survival of the 1906 San Francisco Earthquake. In this section, the results obtained in the foregoing section are reviewed against an estimated pseudoacceleration (PSa) response spectra for single-degree-of-freedom (SDOF) systems with damping ratios ξ of 10 and 5%.

Demand: Calculation of Elastic PSa Spectra

No strong motion record exists of the 1906 San Francisco Earthquake. Hence, to obtain a plausible PSa spectra for this historical earthquake, the following procedure is followed. First, Joyner and Boore's attenuation relationship for western U.S. records is used to compute the probable peak ground acceleration (PGA) of that earthquake, knowing that the moment magnitude of the 1906 San Francisco earthquake has been estimated by others to be 7.9 (Naeim 1989) and that the San Andreas Fault is approximately 15 km from the Montgomery Block building. This gives an estimated PGA of 0.48g. Then, following the proportionality rules of ground spectra proposed by Newmark-Hall, corresponding peak ground velocity (PGV) and peak ground displacement (PGD) of 0.58 m/s and 0.44 m are, respectively, obtained. Finally, the PSa, pseudovelocity (PSV), and displacement (S_d) spectra can be obtained for various damping ratios by multiplying the ground response spectrum by the well-known Newmark-Hall amplification factors (Newmark and Hall 1982). The resulting elastic Newmark-Hall design spectra are used hereafter (personal communication with a prominent practicing engineer actively involved in seismic safety assessments confirmed the foregoing to be a valid approach to obtain a lower bound estimate of the site seismic excitation severity during the 1906 earthquake).

Supply: Ultimate Base-Shear Capacities of Montgomery Block Building

To compare seismic demand (i.e., PSa) with the ultimate structural strength supplied, a rough conversion of previous results is necessary. As described in the foregoing section, maximum base-shear capacities obtained in the previous sections were calculated using material properties at a working stress level. Therefore, these base-shear capacities need to be converted into equivalent ultimate ones. Although not perfectly accurate, ultimate base-shear capacities can be estimated as being equal to three times the previously calculated capacities, because a safety factor of approximately 3.0 is generally assumed in masonry working stress calculations (Seismic Safety Commission 1990) and of 3.3 for the working stress special procedure of the UCBC (Kariotis 1986; Hill 1991).

Then, to locate these capacities on the response spectra, they must be expressed in percentage of gravity (g). For example,

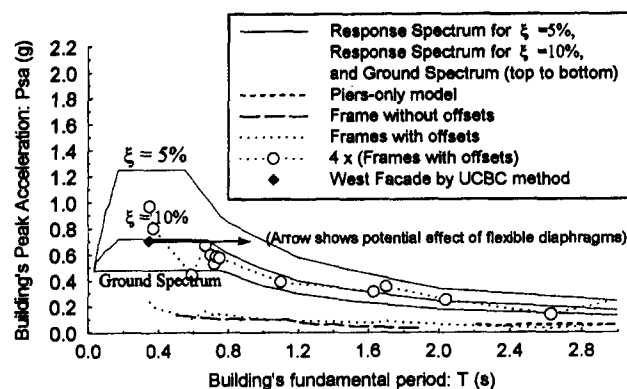


FIG. 9. Pseudospectra Accelerations and Ultimate Capacity Ratios for West Facade

in this section, the west facade was found to have a base-shear capacity of 2% of its reactive tributary supported weight when calculated by the piers-only model. This corresponds to an approximate ultimate strength of 6% of its weight, using the rough safety factor conversion approach described in the foregoing paragraph. Therefore, that model indicates that this facade would remain elastic up to a PSa value of 0.06g, when behaving as a SDOF system.

With the use of that procedure, equivalent ultimate base-shear capacities were calculated for the models considered and results are presented in Fig. 9, expressed in terms of equivalent PSa's. Whenever possible, the results presented in this figure also illustrate the relationship of strength and fundamental period of vibration of the structural models throughout the progression of damage, as described previously.

An analytical method would predict seismic survival of the Montgomery Block building to the 1906 San Francisco Earthquake if it gives an ultimate base-shear capacity in excess of the estimated seismic elastic demand (i.e., response spectra) at any point along the strength-period relationship.

Observations on Supply versus Demand Estimates

As shown in Fig. 9, it appears that none of the analysis methods considered in this limited study can predict seismic survival, with the exception of the UCBC special procedure if compared against the response spectra for 10% damping.

The analysis of the building by the UCBC special procedure has shown that the facade can sustain a maximum base-shear force much greater than that calculated using the frame with rigid offsets model. This is largely attributable to the fact that, in the UCBC special procedure, a rocking mode of behavior that occurs after flexural tensile failure is taken as the resistance limit. Interestingly, this special procedure indicates that a rocking failure mechanism ultimately develops at the fourth story whereas in the frame with rigid offsets model, the failure mechanism occurs at the first story. This difference can be partly explained by recognizing the UCBC's unique assumptions on the vertical distribution of the equivalent static seismic force and on the rocking of cracked piers under rigid continuous and uncracked spandrels.

Also, it must be emphasized that the UCBC procedure does not provide a method to calculate structural period. However, it recognizes the importance of flexible diaphragms in controlling the dynamic behavior of a URM building and, indirectly, that the much longer period of these diaphragms will govern the dynamic response. To reflect this situation, an extension arrow has been drawn in Fig. 9. This may effectively bring the UCBC closer to the 5% damping demand curve, although this aspect was not quantified in this study.

For the frame with rigid offsets model, results indicate that survival would be possible only if the facade was reinforced

to enhance the lateral-load–resistance capacity of this structure by a factor of four. This curve, presented in Fig. 9, follows closely the response spectra for 10% damping. However, such an enhancement is unlikely to have existed in the original building at the time of the earthquake; moreover, the allowable flexural tensile stress “ f_t ” selected for this study is already a high estimate. It is noteworthy that, by taking the allowable flexural stress as infinite (a case of academic interest), shear failure would then govern seismic response and the corresponding ultimate capacity would become $0.67g$ for the frame with rigid offsets model; this is still less than the idealized linear-elastic spectral demands. Hence, according to these more traditional frame models, considerable seismic retrofitting would be required to make this building seismic resistant.

Finally, it remains possible that the Montgomery Block building may have survived the 1906 San Francisco Earthquake for reasons wholly unrelated to the structural considerations in the foregoing paragraph, but rather due to favorable local soil characteristics or other conditions leading to lower seismic excitations at the site than considered probable in this study. Nonetheless, it remains that, in the perspective of a structural seismic vulnerability assessment for exposure to an earthquake risk comparable with the spectra presented in this section, the various models and analysis techniques commonly used by practicing structural engineers and considered in this study would have led to significantly different conclusions regarding the chances of seismic survival and possibly much different seismic rehabilitation measures.

OTHER CONSIDERATIONS

Finite Element Analyses

As numerous structural engineers perceive the finite element method as being the most accurate analysis tool currently available, this method also was used to analyze the Montgomery Block building. Both linear-elastic and nonlinear crack-propagation models (smeared crack models) have been considered using the commercially available program ADINA (Bathe 1995). A subfacade consisting of the center six piers of the original west facade was considered to overcome hardware restrictions of the available workstation. To ensure proper behavior of the model, vertical forces that varied in proportion to the applied lateral loads were added at each pier, at each floor level, to correct the error in overturning moment magnitude that would otherwise result due to the difference in length between the subfacade model and the real facade (Bruneau et al. 1994). Two-dimensional nine-node plane stress rectangular finite elements were consistently used. Smeared crack element shear stiffness reduction factor of 0.5 was used in all nonlinear analyses.

Linear structural finite elements analysis conducted on the facade confirmed that first cracking occurred at the same corresponding base shear as calculated by the frame with rigid offset model. Stress vectors and stress contours plots (Fig. 10) illustrate well the stress-concentration condition that occurred at the corners of every opening, especially at the first floor, and the compression strut that developed across the diagonal of the relatively deep spandrels when the facade was subjected to lateral loads.

The postcracking nonlinear analysis revealed that cracks progressively developed on the first, second, and third floors, mostly at the corners of the spandrels where tensile stress concentration occurred. Once cracks appeared at the foundation, the structure reached its ultimate base-shear capacity. Typically, this maximum capacity was reached at approximately 2.5 times the first cracking load (Fig. 11).

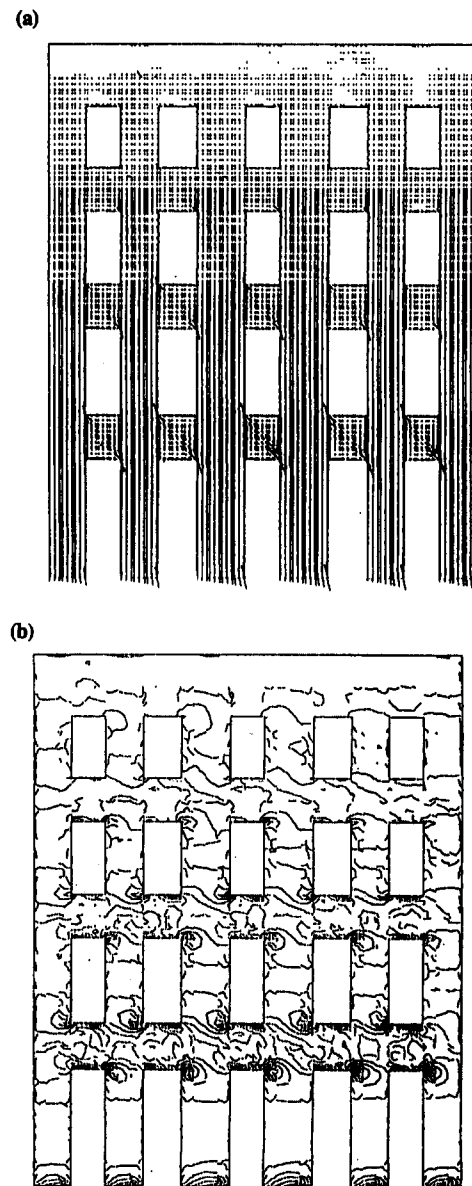


FIG. 10. Linear Finite Element Analysis Results for Montgomery Block Building Subfacade: (a) Stress Vectors of Principal Stresses; (b) Maximum Stress Contours

Based on the foregoing observations and limited investigation, it appears that results from finite element nonlinear cracking analysis would fall approximately halfway between the UCBC results and those for frames with rigid offset. Hence, there remains a significant gap between the base-shear capacities calculated using a stiffness-based analysis procedure, even one that allows the consideration of crack propagation such as the smeared crack finite element analysis and those obtained following a global rigid-body stability analysis strategy such as the one adopted by the special procedure of the UCBC.

Obviously, nonlinear smear-crack analyses are sensitive to variations in model parameters and suffer from a lack of mesh objectivity [e.g., Bhattacharjee and Léger (1993)]. Moreover, special cracking models implemented in commercially available computer programs can be extremely taxing on the hardware requirements and are sometimes prone to numerical stability problems, which was found to be particularly true when entering the strength-degradation range beyond ultimate capacity for this highly perforated URM facade. As a result, although advanced nonlinear cracking finite element analysis

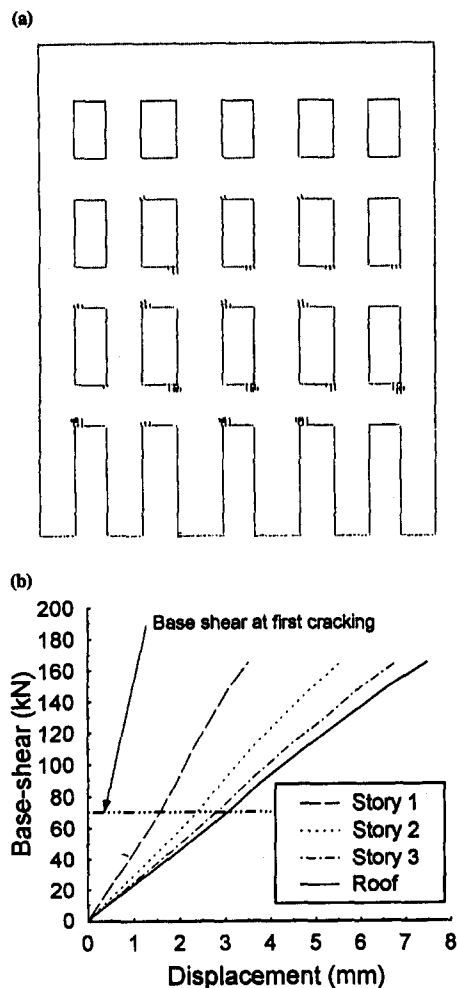


FIG. 11. Smeared-Crack Finite Element Analysis of Montgomery Block Building Subfacade: (a) Crack Propagation Prior to Failure; (b) Calculated Story Displacements

methods have advantages over the simple design methods, these analysis strategies do not seem to have matured to the level of sturdiness and reliability required to be readily used by structural engineers in their everyday practice for this type of problem.

Effect of Diaphragm Flexibility

In anticipation that some engineers might conjecture that the abysmally low seismic base-shear capacities obtained using the traditional design methods are a consequence of poor modeling of the floor and roof diaphragm flexibilities and suggest that the consideration of other diaphragm flexibilities might have greatly and positively affected the seismic resistance of this building, rigid diaphragms have also been considered in separate analyses. The writers doubt that the actual diaphragms of that building could have behaved rigidly. However, the consideration of perfectly rigid and flexible diaphragms was seen as possibly providing bounds on solutions obtained by traditional analytical methods for various diaphragm flexibilities.

The three-dimensional rigid-diaphragm structural analysis of the Montgomery Block building was performed using the piers-only model and the frame model with rigid offsets. As expected for this type of analysis, torsional response developed and all lateral-load-resisting structural elements contributed to seismic resistance. This was particularly noticeable when structural elements failed during the multistep analysis process described earlier, because the loss of resistance caused by the

failure of weaker elements (such as facades) was picked up by stronger structural elements often located in a different vertical plan elsewhere throughout the building.

For the piers-only analyses conducted, flexural tensile failure again governed. However, the maximum base-shear capacity was obtained when first cracking occurred in interior walls located some distance from the facade. After cracking of these walls, the structure became extremely torsionally flexible and its base-shear capacity dropped significantly. The next elements to fail were rigid walls/piers perpendicular to the direction of the applied load. Details of the analysis and results (as well as free-body diagrams of the shear resistance at various stories) are presented elsewhere (Boussabah 1993; Bruneau et al. 1994). However, it is noteworthy that the west facade in this model was too flexible to significantly contribute to the global lateral-load resistance.

For the analyses of frame with rigid offsets, 835 structural elements were needed to model the complete building. Following the same multistep analysis procedure described earlier, capacity of the structure was found to increase progressively even though elements located in the west facade ruptured sequentially. However, this loss of lateral-load resistance of the west facade was observed to be of little significance to the global stiffness and strength of the structure and the maximum base-shear resistance was eventually reached when failure occurred at the same interior wall that failed in the first analysis step of the piers-only model.

As a result of these analyses, base-shear capacities normalized by weight V_{max}/W of 5.2% and 3.3% were obtained for the frame model and piers-only model, respectively. Both values were actually lower than those obtained previously for the west facade alone, simply because the increase in mass (total building mass was used for the rigid-diaphragm analyses) was more considerable than the corresponding increase in strength when considering the rigid-diaphragm building response. However, it is noteworthy that the base-shear capacity obtained using the frame model was only 60% higher than that of piers-only model; the dominance of numerous internal walls on the building's behavior accounts for this lower difference when comparing with the results for the west facade only.

Based on the above, the consideration of a nonflexible diaphragm was deemed unlikely to enhance the seismic base-shear capacity of this building as calculated using traditional piers/frames models.

CONCLUSIONS

In this case study, the maximum seismic capacity of the Montgomery Block building (a URM building that survived the 1906 San Francisco Earthquake) has been calculated using a number of different modeling procedures commonly used by practicing engineers throughout North America when conducting seismic evaluation work. The impact of engineering modeling assumptions on this calculated seismic resistance is found to be considerable, of nearly an order of magnitude in some cases, depending on the structural model selected by the engineer. In fact, in the perspective of a structural seismic vulnerability assessment, if an engineer was asked in 1905 (i.e., prior to the 1906 earthquake) to evaluate the seismic adequacy of the Montgomery Block building using 1990s analytical tools, he might have decided on either of the following: (1) The urgent necessity to perform a comprehensive seismic retrofit of this building, when relying on results obtained by simple or advanced stiffness-based structural analysis models; or (2) the need to simply anchor all walls to the roof and floors to provide structural integrity, based on results from a special procedure relying on rigid-block dynamic stability concepts and recently developed for the seismic evaluation of URM buildings.

Generally, added modeling complexity seemed beneficial as the most simplistic models led to the most conservative predicted seismic resistance, although, as demonstrated by the UCBC analyses, a conceptually more sophisticated model does not necessarily translate into computational difficulties. The piers-only model gave the lowest calculated estimate of seismic resistance, as expressed by base-shear capacities, followed by basic frame models, frame models considering rigid offsets at joints, crack-propagation finite element analysis strategies, and the special procedure of the UCBC, in order of increasing capacity. It is noteworthy that the crack-propagation finite element analyses were difficult to use and have apparently not matured to the level of sturdiness and reliability required to be readily used by practicing engineers for this type of problem. Also, the conventional frame models considered, although not providing the highest estimates of seismic resistance, were useful in demonstrating the potentially beneficial period elongation, which accompanies structural strength degradation as damage progresses, and the lateral load versus displacement behavior resulting from extended multistep analyses using this simplified model illustrated how masonry buildings can show a propensity to sustain large displacements and damage before the development of a complete structural failure mechanism, behaving essentially in a semiductile manner.

In a different perspective, knowing that the Montgomery Block building survived undamaged by the 1906 San Francisco Earthquake, supply versus demand comparisons were attempted using a plausible PSa spectra constructed for that historical earthquake. Results obtained clearly indicate that the onset of cracking is too restrictive a criteria to be used in the seismic evaluation of URM buildings. Indeed, of all the conventional modeling procedures considered, it was found that only the special procedure of the UCBC came close to predicting the observed seismic response. Although keeping in mind the inherent uncertainty in estimating the demand for an historical event for which no strong-motion records exist and the fact that survival of the Montgomery Block building may be wholly attributable to some fortuitous nonstructural circumstances, it remains that the evidence provided in this paper will be valuable to practicing engineers who are still relying on conservative design methods to perform seismic assessment of existing URM structures in those regions of eastern North America that do not have a seismic ordinance endorsing the special procedure of the UCBC.

Additional experimental research is desirable to resolve some of the outstanding issues regarding the credibility of some of the aforementioned models for masonry buildings. Also, a simple computer program for the evaluation of URM structures would definitely be valuable to practicing engineers, but research is needed on how to best implement the fundamental behavior concepts underlying the UCBC procedure into such a program. Testing of complex structural wall configurations also is needed to calibrate such a model and validate the resulting computer program.

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